Girraween High School

Year 12 Mathematics Extension 1

2025 Task 3

Student Number _____

Question	Mark	ME11-3	ME11-4	ME-12-1	ME12-4	MA12-7
1	/1			✓		✓
2	/1	✓				✓
3	/1				✓	✓
4	/1			✓		✓
5	/1				✓	✓
6	/9			✓		✓
7a and 7b	/6	✓				✓
7c	/4			✓		✓
8	/9		✓			✓
9	/13				✓	✓
10	/9			✓		✓
11a	/2			✓		✓
11b	/3				✓	✓
11c	/5	✓				✓
Total	/65	/12	/9	/26	/18	/65

Year 11 Mathematics Extension 1 outcomes

A student:

ME11-3 applies concepts and techniques of inverse trigonometric functions and simplifying expressions involving compound angles in the solution of problems

ME11-4 applies understanding of the concept of a derivative in the solution of problems, including rates of change, exponential growth and decay and related rates of change

Year 12 Mathematics Extension 1 outcomes

A student:

ME12-1 applies techniques involving proof or calculus to model and solve problems

ME12-2 applies concepts and techniques involving vectors and projectiles to solve problems

ME12-3 applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations

ME12-4 uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution

ME12-5 applies appropriate statistical processes to present, analyse and interpret data

ME12-6 chooses and uses appropriate technology to solve problems in a range of contexts

ME12-7 evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms

2025 Year 12 Task 3

Mathematics Extension 1

Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ⬭ B ⬬ C ⬭ D ⬭

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ⬬ B ⬬ C ⬭ D ⬭

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word ***correct*** and drawing an arrow as follows.

correct

A ⬬ B ⬬ C ⬭ D ⬭

1.	A	○	B	○	C	○	D ○
2.	A	○	B	○	C	○	D ○
3.	A	○	B	○	C	○	D ○
4.	A	○	B	○	C	○	D ○
5.	A	○	B	○	C	○	D ○

Girraween High School

2025 Year 12 Task 3

Mathematics Extension 1

General Instructions

- Reading time: 5 minutes
- Working time: 90 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- Answer multiple-choice questions on the multiple-choice answer sheet provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 65

Section I 5 Marks

- Attempt Questions 1 -5
- Allow about 10 minutes for this section

Section II 60 Marks

- Attempt Questions 6 - 11
- Allow about 80 minutes for this section

Section I

5 marks
Attempt Questions 1 – 5
Allow about 10 minutes for this section
Use the multiple-choice answer sheet for Questions 1 – 5

Question 1

Using the substitution $u = \cos x$, or otherwise, find $\int \sin x \cos^2 x \, dx$.

A) $2 \sin x \cos x + c$

B) $\cos^3 x + c$

C) $\frac{1}{3}\cos^3 x + c$

D) $-\frac{1}{3}\cos^3 x + c$

Question 2

A function is defined by the following rule:

$$f(x) = \begin{cases} \sin^{-1} x & \text{for } -1 \leq x < 0 \\ \cos^{-1} x & \text{for } 0 \leq x \leq 1 \end{cases}$$

What is the value of $f\left(-\frac{1}{2}\right) + f(0)$?

A) $-\frac{\pi}{6}$

B) $\frac{\pi}{6}$

C) $\frac{\pi}{3}$

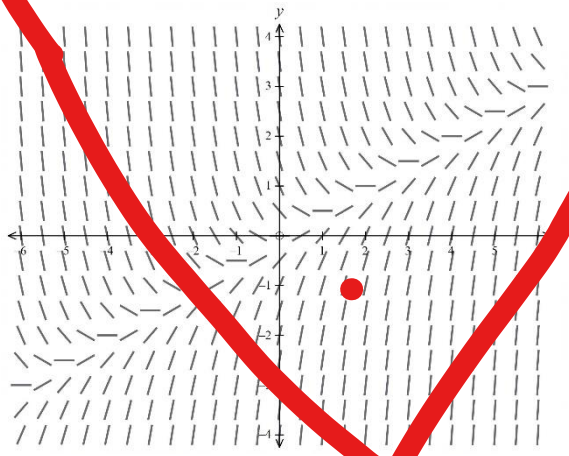
D) $\frac{2\pi}{3}$

Examination continues next page…

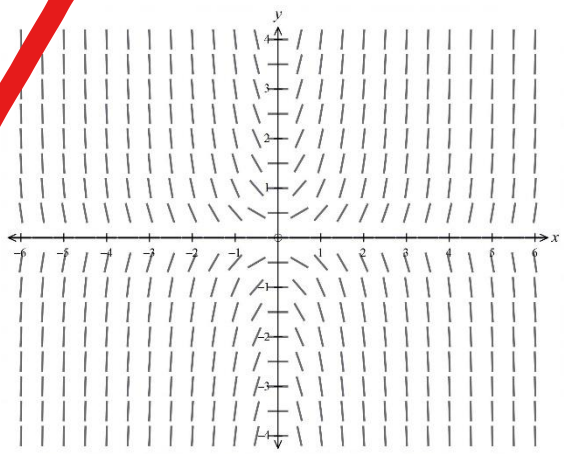
Question 3

Which slope diagram could be used to represent the differential equation $\frac{dy}{dx} = x + 2y$?

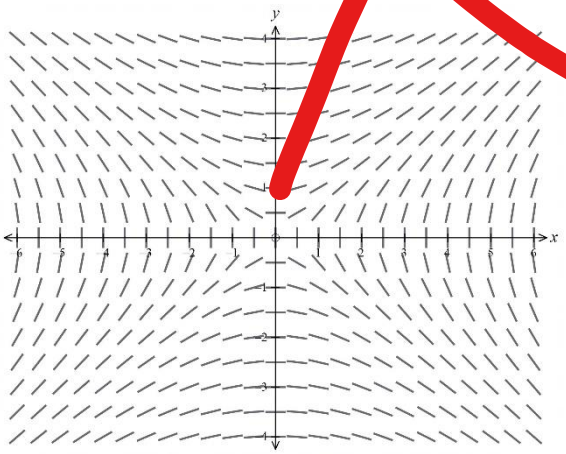
A)



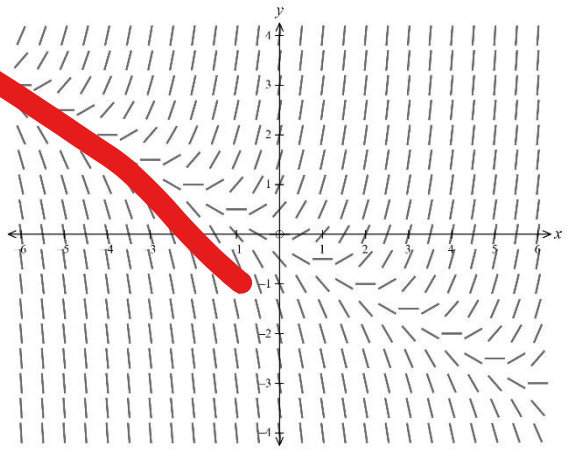
B)



C)



D)



Question 4

Which of the following is equal to $\int_{-1}^{1} \cos^{-1} x \ \ dx$?

A) $\frac{\pi}{3}$

B) $\frac{\pi}{2}$

C) π

D) 2π

Examination continues next page…

Question 5

What are the values of p for which $y = e^{px}$ satisfies the differential equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12y = 0?$$

A) $p = -4, p = 3$

B) $p = -3, p = 4$

C) $p = \pm 3$

D) $p = \pm 4$

Examination continues next page…

Section II

60 marks
Attempt Questions 6 - 11
Allow about 80 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (9 marks)

a) Find $\int \cos^2 5x \ dx$ 2

b) Find $\int \tan^2 2x \ dx$ 2

c) Use the substitution $x = u^2 - 1$, $u \geq 0$, to evaluate $\int_0^3 \frac{x}{\sqrt{x+1}} \ dx$ 3

d) Find $\frac{d}{dx}\cos^{-1}(\sin x)$ 2

Examination continues next page…

Question 7 (10 marks)

a) Sketch the function $y = 4\cos^{-1} 3x$ 3

b) Consider the function $y = \sin(\sin^{-1} x)$

i) State the domain and range of $y = \sin(\sin^{-1} x)$ 2

ii) Hence sketch $y = \sin(\sin^{-1} x)$ 1

c) i) Find $\frac{d}{dx}(x\tan^{-1} x)$ 1

ii) Hence find the exact value of $\int_0^1 \tan^{-1} x \ dx$ 3

Examination continues next page…

Question 8 (9 marks)

a) Newton's Law of Cooling states that the rate of change in the temperature, T^o, of a body is proportional to the difference between the temperature of the body and the surrounding temperature, P^o.

i) If A and k are constants, show that the $T = P + Ae^{kt}$ satisfies the equation $\frac{dT}{dt} = k(T - P)$. 1

ii) A cup of tea with a temperature of 100^oC is too hot to drink. Two minutes later, the temperature has dropped to 93^oC. If the surrounding temperature is 23^oC, calculate A and k. 2

iii) The tea will be drinkable when the temperature has dropped to 80^oC. 2
How long, to the nearest minute, will this take?

b) Michelle is standing at a point *M*, watching a balloon being released from a point *W* which is 100 metres away on the horizontal ground. The balloon rises vertically at a constant velocity of 5m/s.

Let θ radians be the angle of elevation of the balloon at time t seconds and x metres be the distance the balloon has travelled in that time.

i) Find the expression for the rate of change of the angle of elevation of the. balloon over time. Answer in terms of x. 3

ii) Find the rate of change of the angle of elevation over time of the balloon when $\theta = \frac{\pi}{4}$. 1

Examination continues next page…

Question 9 (13 marks)

a) Find the solution to the differential equation $\frac{dy}{dx} = 3x^2y$ given the point (0, 1) lies on the function and $y \neq 0$. 3

b) At time t years the number N of individuals in a population is given by $N = \frac{a}{1+be^{-t}}$ for some constants $a > 0$ and $b > 0$. The initial population size is 20 and the limiting population size is 100.

i) Show that $\frac{dN}{dt} = N(1 - \frac{N}{a})$ 2

ii) Find the values of a and b. 2

c) The proportion, P, of people who have watched a viral video on social media can be modelled by the differential equation $\frac{dP}{dt} = 2P(1 - P)$ where t is the time in weeks.

Initially only 10% of people have watched the video.

i) Show that $\frac{1}{P} + \frac{1}{1-P} = \frac{1}{P(1-P)}$ 1

ii) Hence by solving $\frac{dP}{dt} = 2P(1 - P)$, show that $P = \frac{1}{9e^{-2t} + 1}$ 3

iii) Hence find the exact time taken for half of the people to watch the video. 2

Leave your answer in simplest exact form.

Examination continues next page…

Question 10 (9 marks)

Consider the function $f(x) = \sin^{-1}(\frac{x^2-1}{x^2+1})$.

a) Find the domain of $f(x)$. 1

b) Show that $f'(x) = \frac{2x}{|x|(x^2+1)}$ 2

c) Draw a neat sketch of $y = f(x)$, indicating any intercepts with the axes and any asymptotes. Use about half a page for your graph. 3

d) Restrict the domain of $y = f(x)$ to be $x \geq 0$. Let $y = f^{-1}(x)$ be the inverse function of $y = f(x)$ for this domain. 1

Sketch $y = f^{-1}(x)$ on your graph in part c), labelling the curve $y = f^{-1}(x)$.

e) Find the equation of $f^{-1}(x)$. 2

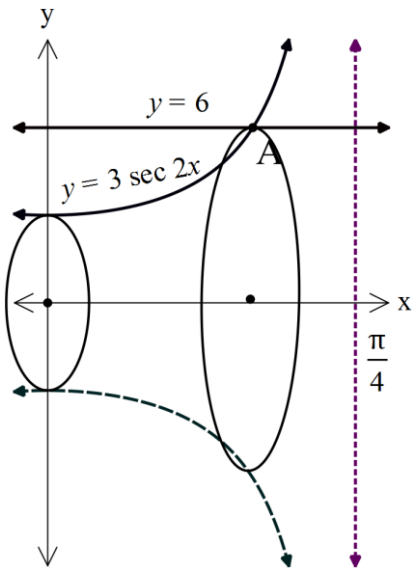
Examination continues next page…

Question 11 (10 marks)

a) Evaluate $\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{16-9x^2}}$ 2

b) The region enclosed between the curve $y = 3\sec 2x$ and the x axis between $x = 0$ and $x = \frac{\pi}{6}$ is rotated about the x axis. 3

Find the volume of the solid formed.



c) i) Show that $\tan^{-1}(n+1) - \tan^{-1}(n-1) = \tan^{-1}(\frac{2}{n^2})$ for $n \geq 1$. 2

ii) Hence, or otherwise, show that 3

$$\sum_{r=1}^{n} \tan^{-1}\frac{2}{r^2} = \tan^{-1}\frac{2n+1}{1-n-n^2} - \frac{\pi}{4}$$

End of Examination.

Mathematics Assessment Solutions

Girraween High School

☐ Year 11	☑ Extension 1	Year 2025
☑ Year 12	☐ Extension 2	Task Number 3

Suggested solutions	Marker's Feedback

Q1 $\int \sin x \cos^2 x \, dx$ let $u = \cos x$

$du = -\sin x$

$-du = \sin x \, dx$

$= -\int u^2 \, du$

$= -\frac{u^3}{3} + C$

$= -\frac{\cos^3 x}{3} + C$ ∴ (D)

Reverse chain rule can also be used.

Q2 $f\left(\frac{1}{2}\right) + f(0)$

$= \sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}(0)$

$= -\frac{\pi}{6} + \frac{\pi}{2}$

$= \frac{\pi}{3}$ (C)

Remember to ignore Questions 3,5 and 9

Suggested solutions	Marker's Feedback

Q3

$\frac{dy}{dx} = x + 2y$

If $x > 0$ & $y > 0$ $\frac{dy}{dx} > 0$

∴ gradient is positive in quad one.

∴ eliminate A

In quad 3 $x < 0, y < 0$

∴ $\frac{dy}{dx} < 0$

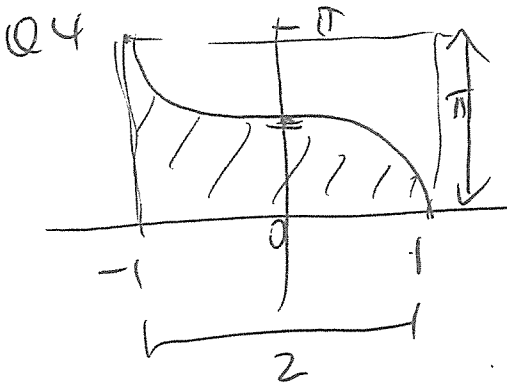
∴ eliminate C

Test point $(2, -1)$

$\frac{dy}{dx} = 2 + 2(-1) = 0$

∴ Answer is (D)

Q4



shaded area

$= \frac{1}{2} \times$ rectangle

$= \frac{1}{2} \times 2 \times \pi$

$= \pi$

$\therefore \int_{-1}^{1} \cos \frac{1}{2}x \, dx = \pi$ (C)

Suggested solutions	Marker's Feedback
(c) $\frac{dy}{dx} = pe^{px}$ $\frac{d^2y}{dx^2} = p^2e^{px}$ $\therefore p^2e^{px} - pe^{px} - 12e^{px} = 0$ $e^{px}(p^2 - p - 12) = 0$ $e^{px}(p-4)(p+3) = 0$ $p = 4, -3$ $\therefore$ (3)	

Suggested solutions	Marker's Feedback

Q6

a) $\int \cos^5 x \, dx$

$= \frac{1}{2} \int (1 + \cos 10x) \, dx$ ① applies identity

$= \frac{1}{2} \left(x + \frac{1}{10} \sin 10x \right) + C$

$= \frac{x}{2} + \frac{1}{20} \sin 10x + C$ ① correct answer

Done reasonably well.

b) $\int \tan^2 2x \, dx$

$= \int (\sec^2 2x - 1) \, dx$ ① applies identity

$= \frac{\tan 2x}{2} - x + C$ ① correct answer

~~A few~~ Few students had the identity $\sec^2 x = 1 + \tan^2 x$ wrong.

∴ the final answer was wrong.

Suggested solutions	Marker's Feedback

Q6

c) $\int_0^3 \frac{x}{\sqrt{x+1}}\,dx$ let $x = u^2 - 1$

$dx = 2u\,du$ Done well.

$x = 0 \to u = 1$

$x = 3 \to u = 2$ A few had limits wrong.

$u^2 = x + 1$

$u = \sqrt{x+1}$ $u \geq 0$

$\int_1^2 \frac{u^2 - 1}{u} 2u\,du$ ① change of variables & substitution

$= 2\int_1^2 u^2 - 1\,du$

$= 2\left[\frac{u^3}{3} - u\right]_1^2$ ① integration

$= 2\left[\frac{2^3}{3} - 2 - \left(\frac{1}{3} - 1\right)\right]$ some had calculation error.

$= \frac{8}{3}$ ① evaluation

Suggested solutions	Marker's Feedback
Q6 d) $\frac{d}{dx} \cos^{-1}(\sin x)$ $= \frac{-1}{\sqrt{1-(\sin x)^2}} \times \cos x$ $= \frac{-\cos x}{\sqrt{\cos^2 x}} = \frac{-\cos x}{\lvert\cos x\rvert}$ $= \frac{-\cos x}{\pm \cos x}$ $= \pm 1$ A lot of students lend only -1 as answer ∴ lost one mark.	① differentiates ① correct answer with cases

Suggested solutions	Marker's Feedback

Q7 a) $y = 4\cos^{-1} 3x$

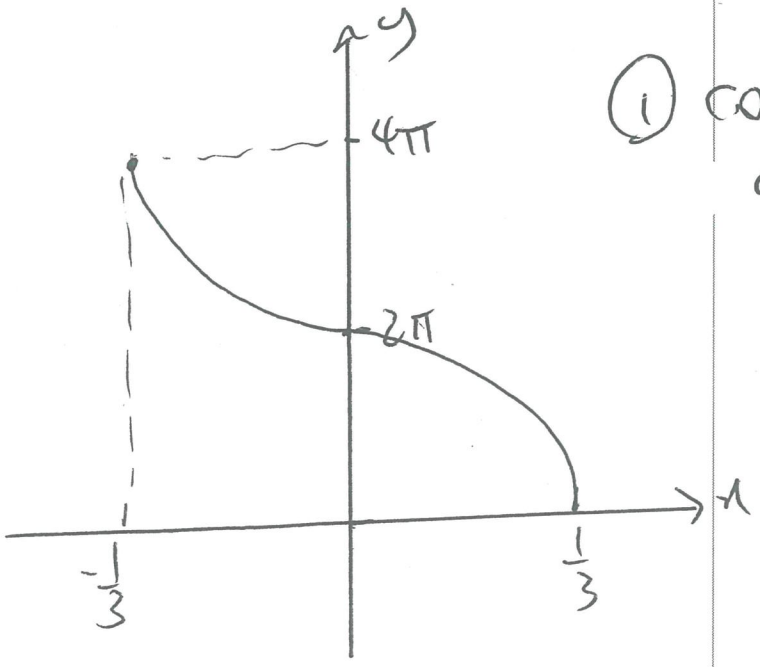
$-1 \leq 3x \leq 1$

D: $-\frac{1}{3} \leq x \leq \frac{1}{3}$ (1)

Mostly done well. Some students did domain or range wrong.

$0 \leq \frac{y}{4} \leq \pi$

Range $0 \leq y \leq 4\pi$ (1)



(1) correct shape and y intercept

Suggested solutions	Marker's Feedback

Q7 b)

i) For $y = \sin(\sin^{-1} x)$

The domain of $\sin^{-1} x$ is

$-1 \leq x \leq 1$.

∴ the domain of $y = \sin(\sin^{-1} x)$ is

$-1 \leq x \leq 1$ (1)

The range of $\sin^{-1} x$ is

$-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$

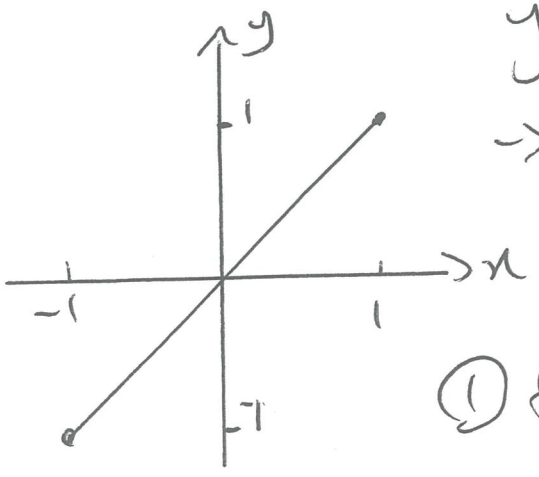
$\therefore \sin\left(-\frac{\pi}{2}\right) \leq y \leq \sin\frac{\pi}{2}$

Range $-1 \leq y \leq 1$. (1)

Majority did well.

Some did do it wrong.

ii)



$y = \sin(\sin^{-1} x)$

$\rightarrow \sin^{-1} y = \sin^{-1} x$

$\therefore y = x$

(1) for correct graph

Suggested solutions	Marker's Feedback

Q7 c)

i) $\frac{d}{dx} x \tan^{-1} x$

$= 1 \cdot \tan^{-1} x + x \frac{1}{1+x^2}$ Done well.

$= \tan^{-1} x + \frac{x}{1+x^2}$ (1)

ii) $\frac{d}{dx} x \tan^{-1} x = \tan^{-1} x + \frac{x}{1+x^2}$ Some students did not show clear working out

$\tan^{-1} x = \frac{d}{dx} x \tan^{-1} x - \frac{x}{1+x^2}$

$\int_0^1 \tan^{-1} x = \left[x \tan^{-1} x \right]_0^1 - \int_0^1 \frac{x}{1+x^2} dx$ (1)

$= (\tan^{-1} 1 - 0) - \frac{1}{2} \left[\ln (1+x^2) \right]_0^1$ (1)

$= \frac{\pi}{4} - \frac{1}{2} (\ln 2 - \ln 1)$

$= \frac{\pi}{4} - \frac{1}{2} \ln 2$ (1)

Suggested solutions	Marker's Feedback

Q8 a) i) $T = P + Ae^{kt}$

$\frac{dT}{dt} = kAe^{kt}$ but $T - P = Ae^{kt}$ ①

$= k(T - P)$ Mostly well done.

(1) A very small minority of students DIDN'T state $T - P = Ae^{kt}$

(2) Some students DERIVED the formula from $\frac{dT}{dt} = k(T-P)$ → They still got the mark BUT need to be told the difference between SHOW & DERIVE.

ii) at $t = 0$ $T = 100$, $P = 23$

$100 = 23 + Ae^0$ ①

$\therefore A = 77$

at $t = 2$ $T = 93$

$93 = 23 + 77e^{k \times 2}$

$70 = 77e^{2k}$

$\ln \frac{10}{11} = \ln e^{2k}$

$\ln \frac{10}{11} = 2k \ln e$

$k = \frac{1}{2} \ln \frac{10}{11}$ ①

(ii) Mainly VERY well done.

Suggested solutions	Marker's Feedback

Q8 a)

(ii) $80 = 23 + 77e^{kt}$

$57 = 77e^{kt}$

$\ln \frac{57}{77} = \ln e^{kt}$

$\ln \frac{57}{77} = kt \ln e$

$t = \frac{1}{k} \ln \frac{57}{77}$

$t = \frac{1}{\frac{1}{2}\ln\frac{10}{11}} \ln \frac{57}{77}$

$= 6.311\ldots$

$= 6$ months

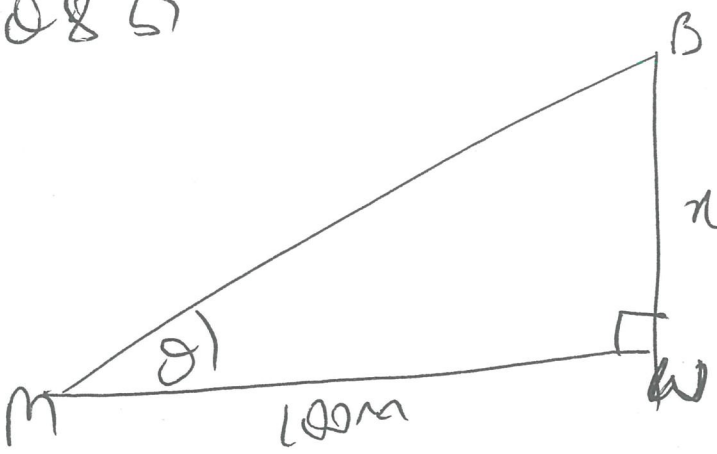
① progress

→ Mainly very well done.

→ Some rounding errors & use of degrees & minutes (NOT penalised) a ①

Suggested solutions	Marker's Feedback

Q8 (i)



$$\frac{dx}{dt} = 5 \text{ m/s}$$

$$\tan\theta = \frac{x}{100}$$

$$x = 100\tan\theta$$

$$\frac{dx}{d\theta} = 100\sec^2\theta \quad \leftarrow (1)_{OR}$$

$$\frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt}$$

$$= \frac{1}{100\sec^2\theta} \times 5$$

$$= \frac{1}{100(1+\tan^2\theta)} \times 5$$

$$= \frac{1}{100\left(1+\frac{x^2}{100^2}\right)} \times 5$$

Or

NOTE MUCH better to start with $\theta = \tan^{-1}\left(\frac{x}{100}\right)$ & differentiate.

Most students who were successful in this question stated this way.

$$\theta = \tan^{-1}\frac{x}{100}$$

$$\frac{d\theta}{dx} = \frac{1}{100}\cdot\frac{1}{1+\left(\frac{x}{100}\right)^2}$$

Better to memorise $\frac{d}{dx}\left(\tan^{-1}\left(\frac{x}{a}\right)\right) = \frac{a}{x^2+a^2}$

~~Wrong formula!~~

A few students started with this method. Most of them couldn't finish by getting $\frac{d\theta}{dt}$ in terms of x.

(1) progress

Suggested solutions	Marker's Feedback

$$\frac{d\theta}{dt} = \frac{1}{100} \cdot \frac{1}{\frac{100^2 + x^2}{100^2}} \times 5$$

$$\frac{d\theta}{dt} = \frac{500}{100^2 + x^2} \quad \text{①}$$

ii) when $\theta = \frac{\pi}{4}$

$x = 100 \tan \frac{\pi}{4}$

$x = 100$

$$\frac{d\theta}{dt} = \frac{500}{100^2 + 100^2} = \frac{1}{40} \text{ radian/sec.} \quad \text{①}$$

Generally well done by those who got these.

MANY students left out units [it wasn't penalised THIS TIME!].

Suggested solutions	Marker's Feedback

Q9(a)(i) $\frac{dy}{dx} = 3x^2y$

$\int \frac{dy}{y} = \int 3x^2\,dx$ ① separates variables

$\log_e|y| = x^3 + c$ ① integrates

Mostly done well. Marks deducted if they wrote 'ln y' instead of 'ln |y|' when integrated the DE.

$y = -e^{x^3+c}$

$y = -e^c e^{x^3}$

$y = Ae^{x^3}$

Sub (0,1)

$1 = Ae^0$

$\therefore A = 1$

$y = e^{x^3}$ ① correct solution

Suggested solutions	Marker's Feedback

Q4 b i)

$$N = \frac{a}{1+be^{-t}} = a(1+be^{-t})^{-1}$$

$$\frac{dN}{dt} = \frac{-a \cdot -be^{-t}}{(1+be^{-t})^2}$$ ① differentiates

$$= \frac{-a}{1+be^{-t}} \cdot \frac{-be^{-t} + 1 - 1}{1+be^{-t}}$$ ①

$$= N\left(1 - \frac{1}{1+be^{-t}}\right)$$

$$= N\left(1 - \frac{N}{a}\right)$$ as required

Generally done well
Some didn't show all necessary working out.

ii) as $t \to \infty$ $e^{-t} \to 0$

$$100 = \frac{a}{1+0}$$

$$\therefore a = 100$$ ①

when $t = 0$, $N = 20$

$$20 = \frac{a}{1+b} = \frac{100}{1+b}$$

$$1+b = 5$$

$$\therefore b = 4$$ ①

Suggested solutions	Marker's Feedback

Q9 c)

i) $\frac{1}{p} + \frac{1}{1-p}$

$= \frac{1-p+p}{p(1-p)}$

$= \frac{1}{p(1-p)}$ = RHS ①

Done well.

ii) $\frac{dp}{dt} = 2p(1-p)$

$\int_{0.1}^{p} \frac{dp}{p(1-p)} = \int_0^t 2\,dt$

At 2 see alternate method

$\int_{0.1}^{p} \left(\frac{1}{p} + \frac{1}{1-p}\right) dp = \int_0^t 2\,dt$ from part (i) ①

$\left[\ln p - \ln(1-p)\right]_{0.1}^{p} = \left[2t\right]_0^t$

$\ln p - \ln(1-p) - \left(\ln 0.1 - \ln(1-0.1)\right) = 2t - 0$

Suggested solutions	Marker's Feedback

q(c)(ii) ctd

$$\ln\frac{p}{1-p} + \ln\frac{q}{1} = 2t$$

(1) significant progress

$$\ln\frac{qp}{1-p} = 2t$$

$$\frac{qp}{1-p} = e^{2t}$$

$$qp = e^{2t}(1-p)$$

$$qp = e^{2t} - pe^{2t}$$

$$qp + pe^{2t} = e^{2t}$$

$$p(q + e^{2t}) = e^{2t}$$

$$p = \frac{e^{2t}}{q + e^{2t}}$$

$$p = \frac{1}{qe^{-2t} + 1}$$

(1) $\div e^{2t}$ $\div e^{2t}$

Generally done well
Some lost marks at the last part of simplification.
* They need to show all working out.

Suggested solutions	Marker's Feedback

9 c) ii) alternate method

$$\int \frac{dp}{p(1-p)} = \int 2\,dt$$

$$\int \frac{1}{p} + \frac{1}{1-p}\,dp = \int 2\,dt$$

$$\ln p - \ln(1-p) = 2t + C$$

when $t = 0$, $p = 0.1$

$$\ln 0.1 - \ln(1-0.1) = 0 + C$$

$$\therefore C = \ln\frac{1}{9}$$

$$\ln p - \ln(1-p) - \ln\frac{1}{9} = 2t \quad (1)$$

$$\ln\frac{9p}{1-p} = 2t$$

etc

Suggested solutions	Marker's Feedback
Q9 c) when $P = \frac{1}{2}$	Mostly done well. Common error was: "$\frac{1}{9e^{-2t}+1} = 50$" instead of "$\frac{1}{9e^{-2t}+1} = \frac{1}{2}$"
$\frac{1}{2} = \frac{1}{9e^{-2t}+1}$	
$2 = 9e^{-2t} + 1$	① progress
$e^{-2t} = \frac{1}{9}$	
$e^{2t} = 9$	
$\ln e^{2t} = \ln 9$	
$2t = \ln 9$	
$t = \frac{1}{2}\ln 9$.	① correct answer

Suggested solutions	Marker's Feedback

Q 10

a) As $x \to \pm\infty$ $\dfrac{x^2-1}{x^2+1} \to 1$

when $x = 0$ $\dfrac{x^2-1}{x^2+1} = -1$

$\therefore -1 \leq \dfrac{x^2-1}{x^2+1} < 1$ for all x.

$\therefore$ Domain of

$\sin^{-1}\left(\dfrac{x^2-1}{x^2+1}\right)$ is all real x.

① bold answer ok.

Many students got 1 mark.

Suggested solutions	Marker's Feedback

Q10

b) $f(x) = \tan^{-1}\left(\frac{x^2-1}{x^2+1}\right)$

$$f'(x) = \frac{1}{\sqrt{1-\left(\frac{x^2-1}{x^2+1}\right)^2}} \times \frac{2x(x^2+1)-2x(x^2-1)}{(x^2+1)^2}$$

(1) applies chain rule and quotient rule

Some are using the strange notation $\left(\frac{x^2-1}{x^2+1}\right)$

$$= \frac{1}{\sqrt{\frac{(x^2+1)^2-(x^2-1)^2}{(x^2+1)^2}}} \times \frac{4x}{(x^2+1)^2}$$

$$= \frac{x^2+1}{\sqrt{x^4+2x^2+1-(x^4-2x+1)}} \times \frac{4x}{(x^2+1)^2}$$

$$= \frac{x^2+1}{\sqrt{4x^2}} \times \frac{4x}{(x^2+1)^2}$$

$$= \frac{2x}{|x|(x^2+1)} \quad * \sqrt{x^2} = |x|$$

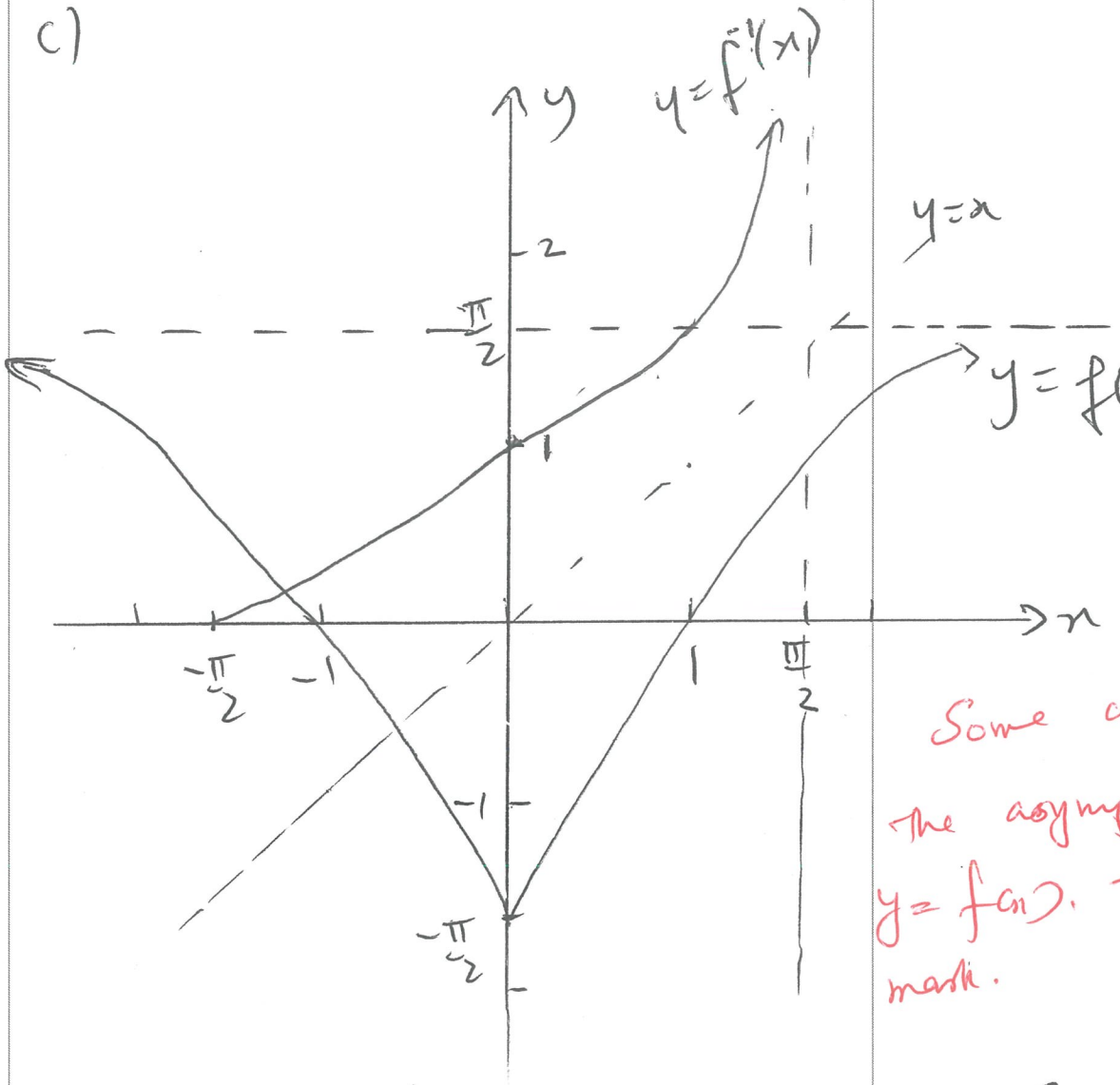
(1)

done well.

Some have difficulty with differentiation.

Q10	Suggested solutions	Marker's Feedback

c)



Some could not get the asymptote of $y = f(x)$. They lost 1 mark.

x intercepts

$f(x) = 0$

$\sin^{-1}\left(\frac{x^2-1}{x^2+1}\right) = 0$

$\therefore x^2 - 1 = 0$

$x = \pm 1$ (1)

asymptote

as $x \to \pm\infty$

$\frac{x^2-1}{x^2+1} \to 1$

$\therefore \sin^{-1}\left(\frac{x^2-1}{x^2+1}\right) \to \frac{\pi}{2}$ (1)

y intercept

let $x = 0$ $\quad y = \sin^{-1}\left(\frac{0-1}{0+1}\right)$

$y = -\frac{\pi}{2}$ (1)

Suggested solutions	Marker's Feedback

e) see graph for d)

① for shape & intercepts

* no penalty for missing asymptote & $y = x$.

f)

$$x = \sin^{-1}\left(\frac{y^2-1}{y^2+1}\right)$$

$$\sin x = \frac{y^2-1}{y^2+1}$$

$$(y^2+1)\sin x = y^2-1$$

$$y^2\sin x + \sin x = y^2 - 1$$

$$y^2(1-\sin x) = 1+\sin x$$

$$y^2 = \frac{1+\sin x}{1-\sin x}$$

$$y = \pm\sqrt{\frac{1+\sin x}{1-\sin x}}$$

① progress

students who forgot to swap x and y were awarded 0 mark.

Suggested solutions	Marker's Feedback
$f^{-1}(x) = \sqrt{\frac{1+\sin x}{1-\sin x}}$ correct answer with justification (1)	Domain of $f(x)$ is $x \geq 0$ $\therefore$ range of $f^{-1}(x)$ is $y \geq 0$ students who wrote $y > 0$ were given mark.

Suggested solutions	Marker's Feedback

Q11 a)

$$\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{16-9x^2}}$$

$$= \int_0^{\frac{2}{3}} \frac{dx}{\sqrt{9\left(\frac{16}{9}-x^2\right)}}$$

$$= \frac{1}{3}\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{\frac{16}{9}-x^2}}$$

$$= \frac{1}{3}\left[\sin^{-1}\frac{3x}{4}\right]_0^{\frac{2}{3}}$$ ① integrates correctly

$$= \frac{1}{3}\left[\sin^{-1}\frac{3}{4}\cdot\frac{2}{3} - \sin^{-1}0\right]$$

$$= \frac{1}{3}\left(\sin^{-1}\frac{1}{2} - 0\right)$$

$$= \frac{1}{3}\cdot\frac{\pi}{6}$$

$$= \frac{\pi}{18}$$ ① evaluates correctly

Generally well done

Suggested solutions	Marker's Feedback

Q11 b)

$$V = \pi \int y^2 \, dx$$

$$= \pi \int_0^{\frac{\pi}{6}} [3\sec 2x]^2 dx$$

① sets up integral

$$= \pi \int_0^{\frac{\pi}{6}} 9 \sec^2 2x \, dx$$

$$= 9\pi \cdot \frac{1}{2} \left[\tan 2x\right]_0^{\frac{\pi}{6}}$$

① correct integration

$$= \frac{9\pi}{2} \left(\tan 2\frac{\pi}{6} - \tan 0\right)$$

$$= \frac{9\pi}{2} \cdot (\sqrt{3} - 0)$$

$$= \frac{9\sqrt{3}\,\pi}{2} \text{ units}^3$$

① correct answer

Very well done.

Suggested solutions	Marker's Feedback

Q11(c) i)

$\tan^{-1}(n+1) - \tan^{-1}(n-1) = \tan^{-1}\left(\frac{2}{n^2}\right)$

$\rightarrow \tan\left[\tan^{-1}(n+1) - \tan^{-1}(n-1)\right] = \frac{2}{n^2}$

let $\alpha = \tan^{-1}(n+1)$ $\quad \beta = \tan^{-1}(n-1)$

$\therefore \tan\alpha = n+1$ $\quad \tan\beta = n-1$

LHS $= \tan\left[\tan^{-1}(n+1) - \tan^{-1}(n-1)\right]$

$= \tan(\alpha - \beta)$

$= \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$= \dfrac{(n+1) - (n-1)}{1 + (n+1)(n-1)}$

$= \dfrac{2}{1 + (n^2 - 1)}$

$= \dfrac{2}{n^2}$

$=$ RHS

① significant progress

well done

① completes solution

Suggested solutions	Marker's Feedback

Q11 c) ii)

RTP $\sum_{r=1}^{n} \tan^{-1}\frac{2}{r^2} = \tan^{-1}\left(\frac{2n+1}{1-n-n^2}\right) - \frac{\pi}{4}$

$LHS = \sum_{r=1}^{n} \tan^{-1}\frac{2}{r^2}$

$= \sum_{r=1}^{n}\left[\tan^{-1}(k+1) - \tan^{-1}(k-1)\right]$ from part (i)

$= \tan^{-1}2 - \tan^{-1}0 + \tan^{-1}3 - \tan^{-1}1$

$+\tan^{-1}4 - \tan^{-1}2 + \cdots$

$\cdots + \tan^{-1}(n-1) - \tan^{-1}(n-3)$

$+ \tan^{-1}n - \tan^{-1}(n-2)$

$+ \tan^{-1}(n+1) - \tan^{-1}(n-1)$

① progress

rearranging

$= [\tan^{-1}2 + \tan^{-1}3 + \tan^{-1}4 + \cdots \tan^{-1}(n+1)]$

$-[\tan^{-1}0 + \tan^{-1}1 + \tan^{-1}2 + \cdots \tan^{-1}(n-1)]$

$+ \tan^{-1}n + \tan^{-1}(n+1)$

after cancelling

$= \tan^{-1}n + \tan^{-1}(n+1) - \tan^{-1}1$

① Significant progress

Suggested solutions	Marker's Feedback

Using the result from (i)

$$\tan\left[\tan^{-1}n + \tan^{-1}(n+1)\right]$$

$$= \frac{n + (n+1)}{1 - n(n+1)}$$

$$= \frac{2n+1}{1-n-n^2}$$

i.e. $\tan^{-1}n + \tan^{-1}(n+1)$

$$= \tan^{-1}\left(\frac{2n+1}{1-n-n^2}\right)$$

} (1) applies result from (i)

$$\therefore \sum_{r=1}^{n} \tan^{-1}\left(\frac{2}{r^2}\right) = \tan^{-1}\left(\frac{2n+1}{1-n-n^2}\right) - \tan^{-1}$$

$$= \tan^{-1}\left(\frac{2n+1}{1-n-n^2}\right) - \frac{\pi}{4}$$

many non attempts but well done by those who attempted it.